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ADAPTIVE TRACKING ALGORITHM FOR TRACK-  
ING AIR TARGETS WITH SEARCH RADARS

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## ADAPTIVE TRACKING ALGORITHM FOR TRACKING AIR TARGETS WITH SEARCH RADARS

### INTRODUCTION

Since the time of the Wiener Filtering Theory, which was based on stationary processes, there has been increased interest in adaptive filters. Some of the better known adaptive systems are adaptive antennas [1], phase-lock loops [2], and the Kalman filter [3]. Recently there has been considerable interest in adaptive tracking systems [4-12]. Most of these adaptive systems use a feedback loop to adjust the parameters in the system. A popular method of adjusting the systems' gains is to use a least mean square (LMS) error criterion and minimize it with respect to the gains [1,4]. This report describes an adaptive  $\alpha - \beta$  filter based on this LMS error criterion. Although its operation is similar in principle to the adaptive antennas [1] and tracking system [4], there are marked differences. We begin by reviewing the  $\alpha - \beta$  filter.

### REVIEW OF THE $\alpha - \beta$ FILTER

The  $\alpha - \beta$  filter is defined as [13,14]

$$\begin{bmatrix} x_s(k) \\ v_s(k) \end{bmatrix} = \begin{bmatrix} (1 - \alpha) & (1 - \alpha)T(k) \\ \frac{-\beta}{T(k)} & (1 - \beta) \end{bmatrix} \begin{bmatrix} x_s(k-1) \\ v_s(k-1) \end{bmatrix} + \begin{bmatrix} \alpha \\ \frac{\beta}{T(k)} \end{bmatrix} [x_m(k)] \quad (1)$$

and

$$x_p(k+1) = [1 \ T(k+1)] \begin{bmatrix} x_s(k) \\ v_s(k) \end{bmatrix} \quad (2)$$

where

$x_s(k)$  = smoothed position

$v_s(k)$  = smoothed velocity

$x_p(k)$  = predicted position

$x_m(k)$  = measured position

$x_m(k) = u(k) + w(k)$

Note: Manuscript submitted July 26, 1974.

B. H. CANTRELI

$u(k)$  = true position (band-limited but unknown)

$w(k)$  = zero mean white Gaussian measurement noise

$T(k)$  = time between samples

$\alpha, \beta$  = system gains.

Applying the  $z$ -transform to Eqs. (1) and (2), we find that the transfer functions of the system are

$$H_x = \frac{x_s(z)}{x_m(z)} = \frac{\alpha z \left[ z + \frac{(\beta - \alpha)}{\alpha} \right]}{z^2 - z(2 - \alpha - \beta) + (1 - \alpha)} \quad (3)$$

$$H_v = \frac{v_s(z)}{x_m(z)} = \frac{\frac{\beta}{T(k)} z(z - 1)}{z^2 - z(2 - \alpha - \beta) + (1 - \alpha)} \quad (4)$$

The transfer functions, Eqs. (3) and (4), are placed into standard notation for a second order system:

$$H(\cdot) = \frac{(\cdot)}{z^2 - 2ze^{-\xi\omega_0 T(k)} \cos \omega_d T(k) + e^{-2\xi\omega_0 T(k)}} \quad (5)$$

Equating terms in the denominators of Eqs. (3) and (4) with that of Eq. (5), we obtain

$$\alpha = 1 - e^{-2\xi\omega_0 T(k)} \quad (6)$$

$$\beta = 1 + e^{-2\xi\omega_0 T(k)} - 2e^{-\xi\omega_0 T(k)} \cos \omega_d T(k), \quad (7)$$

or conversely,

$$\xi = \frac{\ln \frac{1}{\sqrt{1 - \alpha}}}{\sqrt{\left[ \ln \frac{1}{\sqrt{1 - \alpha}} \right]^2 + \left[ \cos^{-1} \left( \frac{(2 - \alpha - \beta)}{2\sqrt{1 - \alpha}} \right) \right]^2}} \quad (8)$$

$$\omega_d = \frac{1}{T(k)} \cos^{-1} \frac{(2 - \alpha - \beta)}{2\sqrt{1 - \alpha}} \quad (9)$$

$$\omega_0 = \frac{\omega_d}{\sqrt{1 - \xi^2}} \quad (10)$$

where  $\xi$ ,  $\omega_d$ , and  $\omega_0$  are the classic damping coefficient, damped natural frequency, and natural frequency of a second order system. In Ref. 13, it was suggested that if one set  $\alpha$ , then  $\beta$  could be found by

$$\beta = \frac{\alpha^2}{(2 - \alpha)} \quad (11)$$

Substituting this expression into Eq. (8), we find that  $\xi$  varies from 0.707 to 0.86 as  $\alpha$  varies from zero to unity. We find that by using Eq. (11), the system remains near critical damping for all values of  $\alpha$  and that  $\alpha$  and  $T(k)$  controls the system bandwidth. This concludes the review of the  $\alpha - \beta$  filter.

### LEAST-MEAN-SQUARE ERROR CRITERION

The least mean square error criterion which is defined as the expected value of the square of the difference between the target's predicted and measured position was chosen as the system's performance measure;

$$\xi = E\{[x_p(k+1) - x_m(k+1)]^2\} \quad (12)$$

The gradient of  $\xi$  with respect to the system's gains is found by interchanging the expected value and derivative operation;

$$\nabla_{\alpha} = E\left\{[x_p(k+1) - x_m(k+1)] \frac{\partial x_p(k+1)}{\partial \alpha}\right\} = 0 \quad (13)$$

$$\nabla_{\beta} = E\left\{[x_p(k+1) - x_m(k+1)] \frac{\partial x_p(k+1)}{\partial \beta}\right\} = 0 \quad (14)$$

The gradients are set to zero and we solve for the system's gains which yield minimum error. If Eq. (1) is substituted into Eq. (2),  $x_p(k+1)$  becomes

$$x_p(k+1) = x_p(k) + T(k+1)v_s(k-1) + \alpha[x_m(k) - x_p(k)] + \frac{\beta T(k+1)[x_m(k) - x_p(k)]}{T(k)} \quad (15)$$

where  $x_p(k) = x_s(k-1) + T(k)v_s(k-1)$ .

The partial derivatives of  $x_p(k+1)$  are proportional to

$$\frac{\partial x_p(k+1)}{\partial \alpha} \sim \frac{\partial x_p(k+1)}{\partial \beta} \sim [x_m(k) - x_p(k)] \quad (16)$$

For notational convenience we define

$$y_1 = [x_p(k) + v_s(k-1)T(k+1) - x_m(k+1)] \quad (17)$$

$$y_2 = [x_m(k) - x_p(k)]. \quad (18)$$

Substituting Eqs. (15) and (16) into Eqs. (13) and (14) and using the definitions Eqs. (17) and (18), we find that Eqs. (13) and (14) are identical and are given by

$$E \left[ y_1 y_2 + \alpha y_2 y_2 + \frac{\beta y_2 y_2 T(k+1)}{T(k)} \right] = 0. \quad (19)$$

The set of Eqs. (13) and (14) are singular. To properly constrain the gradients, we use the relation given by Eq. (11) as  $\beta = \alpha^2 / (2 - \alpha)$ , which yields

$$E \left( \alpha^2 \left\{ y_2 y_2 \left[ \frac{T(k+1)}{T(k)} \right] - y_2 y_2 \right\} + \alpha [2y_2 y_2 - y_2 y_1] + 2y_1 y_2 \right) = 0. \quad (20)$$

Finally, since the statistics of  $y_1$  and  $y_2$  are not known, the expected value will be replaced with a time average:

$$\alpha^2 \left[ y_2 y_2 \frac{T(k+1)}{T(k)} - y_2 y_2 \right] + \alpha [2y_2 y_2 - y_2 y_1] + 2y_1 y_2 = 0. \quad (21)$$

#### ADAPTIVE FILTER WITH UNIFORM UPDATES

For uniform updates  $T(k+1) = T(k) = T$ , and  $\alpha$  directly relates to the bandwidth of the filter. Therefore, Eq. (21) becomes

$$\alpha = \frac{-2\overline{y_1 y_2}}{2\overline{y_2 y_2} - \overline{y_1 y_2}}. \quad (22)$$

Defining  $p_1(k) = \overline{y_1 y_2}$  and  $p_2(k) = \overline{y_2 y_2}$ , we compute  $p_1(k)$  and  $p_2(k)$  by passing  $y_1 y_2$  and  $y_2 y_2$  through low-pass filters;

$$p_1(k) = \mathcal{F}_a p_1(k-1) + (1 - \mathcal{F}_b)(y_1 y_2) \quad (23)$$

$$p_2(k) = \mathcal{F}_b p_2(k-1) + (1 - \mathcal{F}_b)(y_2 y_2). \quad (24)$$

An example is used to illustrate the filter's performance. A target is flown away from the radar at a speed of 1000 ft/s, it makes an 180-degree, 3-g turn, and then flies in a straight line in a crossing course near the radar, as illustrated in Fig. 1. The radar updates

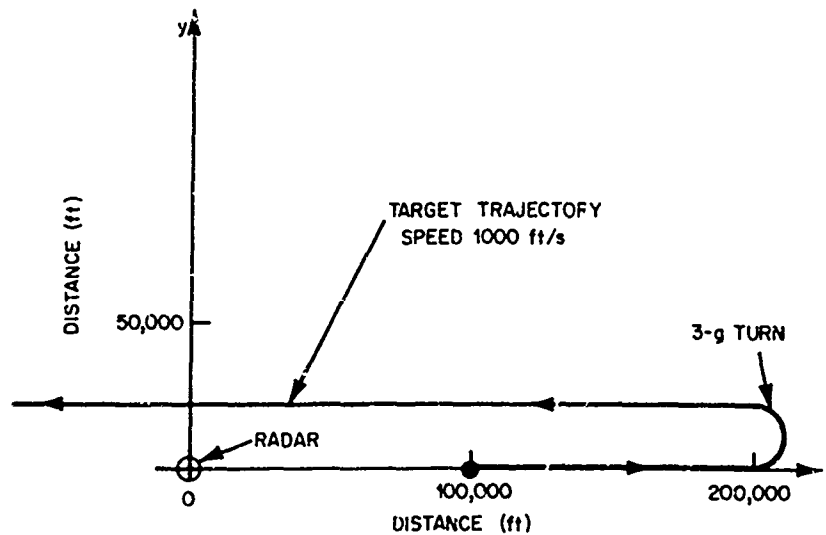


Fig. 1 - Target trajectory used in examples

the track every 4 s. The tracking is performed in range and the range measurement error is assumed to be Gaussian distributed with  $\sigma = 500$  ft. Constants  $\mathcal{I}_a$  and  $\mathcal{I}_b$  were chosen to be 0.819 and 0.91, respectively. The filter's equations are summarized below.

Measure  $x_m(k+1)$

$$p_1(k) = \mathcal{I}_a p_1(k-1) + (1 - \mathcal{I}_b)[x_p(k) + v_s(k-1)T - x_m(k+1)]$$

$$[x_m(k) - x_p(k)]$$

$$p_2(k) = \mathcal{I}_b p_2(k-1) + (1 - \mathcal{I}_b)[x_m(k) - x_p(k)]^2$$

$$\alpha = \frac{-2p_1(k)}{2p_2(k) - p_1(k)}$$

$$\beta = \frac{\alpha^2}{(2 - \alpha)}$$

$$k = k + 1$$

(25)

$$\begin{bmatrix} x_s(k) \\ v_s(k) \end{bmatrix} = \begin{bmatrix} (1 - \alpha) & (1 - \alpha)T \\ -\frac{\beta}{T} & (1 - \beta) \end{bmatrix} \begin{bmatrix} x_s(k-1) \\ v_s(k-1) \end{bmatrix} + \begin{bmatrix} \alpha \\ \frac{\beta}{T} \end{bmatrix} [x_m(k)]$$

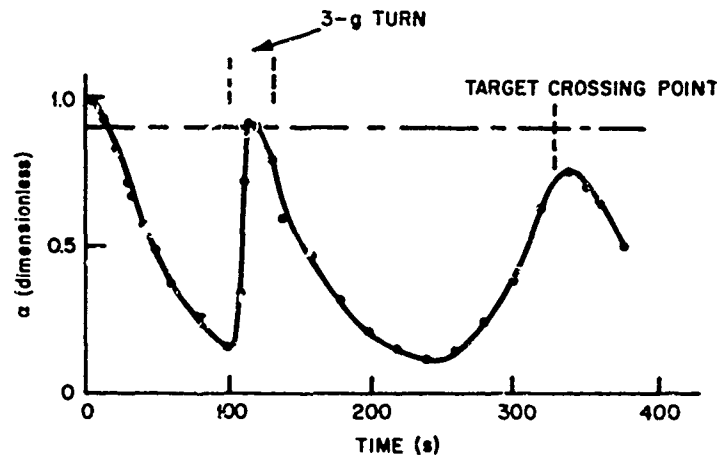
Store  $x_p(k)$ ,  $v_s(k-1)$ , and  $x_m(k)$

$$x_p(k+1) = x_s(k) + Tv_s(k)$$

Repeat.

(25)

The track is initialized by setting all positions equal to the first measured position and the velocity equal to zero. On the next two measured positions the algorithm is operated normally, but we constrain  $\alpha$  and  $\beta$  to be equal to unity. After the first three detections the algorithm is operated normally. The mean values of the bandwidth  $\alpha$  and the error  $x_m(k) - x_p(k)$  are plotted as a function of time in Figs. 2 and 3. We find that the bandwidth decreases when there is only straight-line motion of the target in range and increases to accommodate the target's 3-g turn and accelerations caused by the target's crossing course in front of the radar. The mean error is small except in the regions of high accelerations in which the filter lags the target. The standard deviation of the error decreases as the filter's bandwidth is decreased and increases when the filter's bandwidth increases, which occurs to accommodate the accelerations of the target, as shown in Fig. 4. It was found that the system acted much better when the gain of the low-pass filter Eq. (23) was less than unity ( $\mathcal{F}_a < \mathcal{F}_b$ ).

Fig. 2 — Adaptive adjustment of the parameter  $\alpha$  as a function of time

The adaptive filter was compared to two constant-bandwidth filters of  $\alpha = 0.9$  and  $0.2$  respectively. First, looking at the filter of  $\alpha = 0.2$ , we find that the response is sluggish and the mean errors caused by the high accelerations are enormous. However, the standard deviation does settle to a reasonably small value. The filter with  $\alpha = 0.9$  responds rapidly to the accelerations because of its wide bandwidth, but the standard deviation of the error does not achieve a small value. The adaptive system adapts to the changing environment and in most cases yields reasonably small errors.

#### ADAPTIVE FILTER WITH NONUNIFORM UPDATES

For nonuniform updates  $\alpha$  does not directly relate to the natural frequency  $\omega_0$ , but is related by

$$\alpha = 1 - e^{-2\xi\omega_0 T(k)}. \quad (26)$$

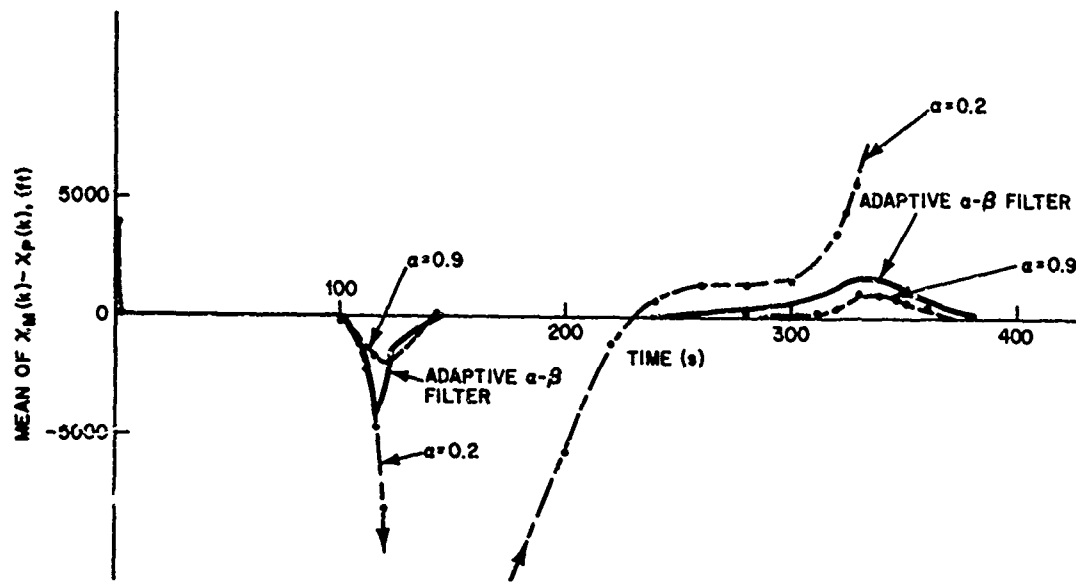


Fig. 3 — Mean error as a function of time for constant gain and adaptive  $\alpha - \beta$  filter operated with constant update time

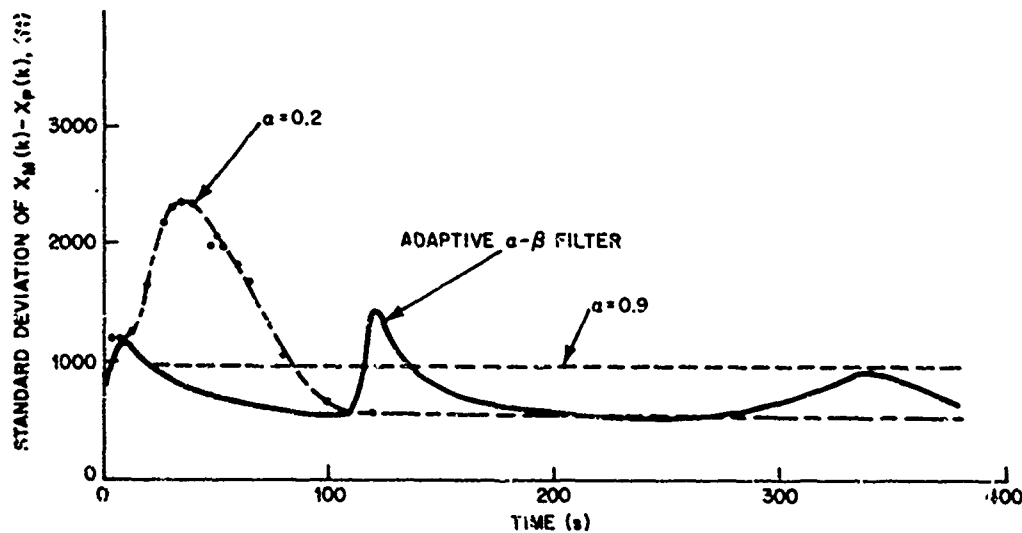


Fig. 4 — Standard deviation of the error as a function of time for constant gain and adaptive filter operated with constant update time

The adjustment of the bandwidth  $\omega_0$  is performed by solving Eq. (21) using the definition Eq. (26). However, the time averages are difficult to perform and yet solve for  $\omega_0$ . We introduce an approximation for Eq. (26),

$$\alpha = \frac{2\xi\omega_0 T(k)}{1 + 2\xi\omega_0 T(k)} \quad (27)$$

Substituting Eq. (27) into Eq. (21), one finds

$$\begin{aligned} \omega_0^2 [\overline{y_2 y_2 T(k) T(k+1)} + \overline{y_2 y_2 T^2(k)} + \overline{y_2 y_1 T^2(k)}] + \frac{0.5\omega_0}{\xi} \\ \times [\overline{2y_2 y_2 T(k)} + \overline{3y_2 y_1 T(k)}] + \frac{0.5}{\xi} [\overline{y_2 y_1}] = 0. \end{aligned} \quad (28)$$

The time averages in Eq. (28) are computed by first-order, low-pass filters, and the adjustable bandwidth is computed by solving the quadratic equation. The target trajectory described in Fig. 1 is used to illustrate the filter performance. The sampling time  $T(k)$  is assumed to be a uniformly distributed random variable that can take on values of 0.5 to 8 s. The filter's equations are summarized below.

$$\begin{aligned} \mathcal{J}_a &= e^{-\omega_a T(k+1)} \\ \mathcal{J}_b &= e^{-\omega_b T(k+1)} \\ q_1(k) &= \mathcal{J}_a q_1(k-1) + (1 - \mathcal{J}_b)[x_p(k) + v_s(k-1)T(k+1) - x_m(k+1)] \\ &\quad \times [x_m(k) - x_p(k)] \\ q_2(k) &= \mathcal{J}_a q_2(k-1) + (1 - \mathcal{J}_b)[x_p(k) + v_s(k-1)T(k+1) - x_m(k+1)] \\ &\quad \times [x_m(k) - x_p(k)] T(k) \\ q_3(k) &= \mathcal{J}_a q_3(k-1) + (1 - \mathcal{J}_b)[x_p(k) + v_s(k-1)T(k+1) - x_m(k+1)] \\ &\quad \times [x_m(k) - x_p(k)] T(k) T(k) \\ q_4(k) &= \mathcal{J}_b q_4(k-1) + (1 - \mathcal{J}_b)[x_m(k) - x_p(k)][x_m(k) - x_p(k)] T(k) \\ q_5(k) &= \mathcal{J}_b q_5(k-1) + (1 - \mathcal{J}_b)[x_m(k) - x_p(k)][x_m(k) - x_p(k)] T(k) T(k) \\ q_6(k) &= \mathcal{J}_b q_5(k-1) + (1 - \mathcal{J}_b)[x_m(k) - x_p(k)][x_m(k) - x_p(k)] T(k) T(k+1) \\ \omega_0^2 [q_6(k) + q_5(k) + q_3(k)] + \omega_0 \frac{1}{2} \xi [2q_4(k) + 3q_2(k)] + \frac{1}{2} \xi q_1(k) &= 0 \\ \alpha &= 1 - e^{-2\xi\omega_0 T(k+1)} \end{aligned} \quad (29)$$

$$\beta = \frac{\alpha^2}{(2 - \alpha)}$$

$$k = k + 1$$

$$\begin{bmatrix} x_s(k) \\ v_s(k) \end{bmatrix} = \begin{bmatrix} (1 - \alpha) & (1 - \alpha)T(k) \\ \frac{-\beta}{T(k)} & (1 - \beta) \end{bmatrix} \begin{bmatrix} x_s(k-1) \\ v_s(k-1) \end{bmatrix} + \begin{bmatrix} \alpha \\ \frac{\beta}{T(k)} \end{bmatrix} [x_m(k)].$$

Store  $x_p(k)$ ,  $v_s(k-1)$ ,  $x_m(k)$ ,  $T(k)$

$$x_p(k+1) = x_s(k) + T(k+1)v_s(k)$$

Repeat.

(29)

Note that the first order averaging filter's time constants  $\mathcal{T}_a$  and  $\mathcal{T}_b$  are adjusted as a function of the sampling interval where  $\omega_a = 0.05$  and  $\omega_b = 0.024$ . This is done to maintain a reasonably constant bandwidth in time.  $\xi$  was set equal to 0.75. The filter was initiated by setting all positions equal to the first measured position and the velocity equal to zero. On the next two measurements  $\alpha$  was adjusted by setting  $\omega_0 = 0.2$ . All subsequent measurements followed the algorithm. Time histories of the bandwidth and error are shown in Figs. 5 and 6 for a given trial. We find that the results are similar to the uniform update cases (Figs. 2 and 3) in that the bandwidth of the filter is wide when the target is accelerating, and decreases to a small value when the target is moving in a straight line.

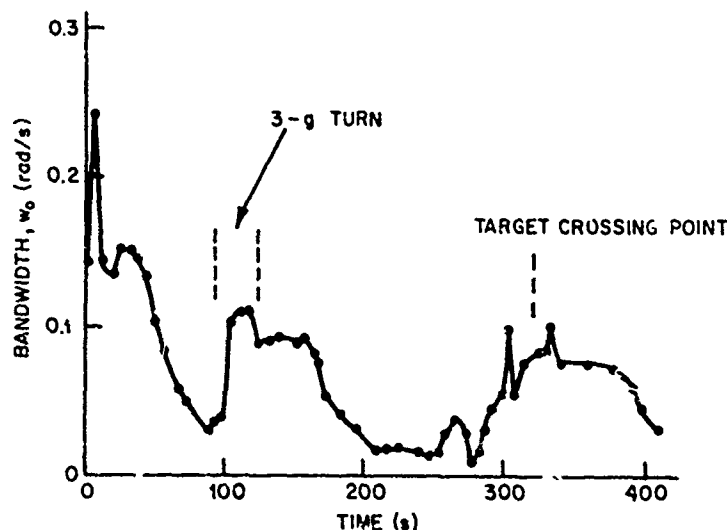


Fig. 5 - Adaptive bandwidth of filter as a function of time for a random update time

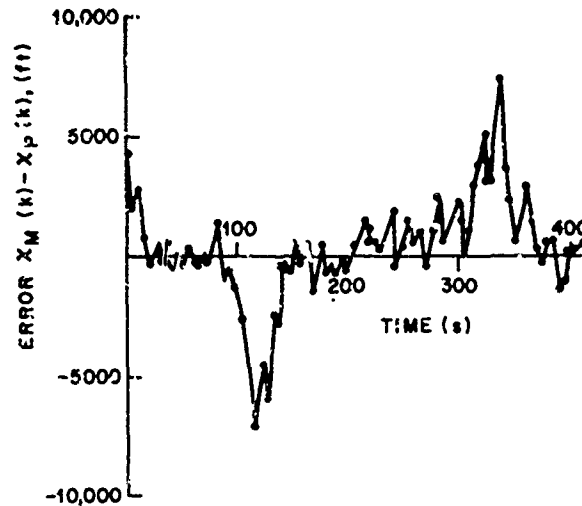


Fig. 6 — Time history of error for adaptive filter operated with a random update time

## SUMMARY

A means of adaptively adjusting the bandwidth in an  $\alpha - \beta$  filter was obtained by using the LMS criteria. The expected value operations were approximated with time averages. An example was performed to show that the bandwidth of the filter decreased when the target was flying a straight-line course and opened up when the target was under acceleration (real and apparent). The filter seemed to respond reasonably rapidly to its changing environment even though the time between samples was quite long.

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